

Role of the $\Delta(1232)$ in pion-deuteron scattering at threshold within chiral effective field theory

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Abstract

We investigate the role of the delta isobar in the reaction $\pi d \rightarrow \pi d$ at threshold in chiral effective field theory. We discuss the corresponding power counting and argue that this calculation completes the evaluation of diagrams up to the order $\chi^{3/2}$, with χ the ratio of the pion to the nucleon mass. The net effect of all delta contributions at this order to the pion-deuteron scattering length is $\delta a_{\pi d}^{\Delta} = (2.4 \pm 0.4) \times 10^{-3} m_{\pi}^{-1}$.

1 Introduction

Chiral perturbation theory (ChPT) is the effective field theory of the Standard Model at low energies allowing for high accuracy calculations of hadronic observables. It is a systematic expansion around the chiral limit (vanishing quark masses) and vanishing external momenta. ChPT can be applied to systems containing pions, nucleons and external sources. Here we focus on the πNN system — in particular the pion–deuteron system at threshold.

In the original formulation, only pions and nucleons appear as dynamical degrees of freedom [1, 2], whereas the impact of baryon resonances as well as heavier mesons is absorbed into certain low-energy constants. From phenomenological studies it is well known that the delta isobar $\Delta(1232)$ plays a very special role in low energy nuclear dynamics [3] as a consequence of the relatively large $\pi N \Delta$ coupling and the quite small delta–nucleon mass difference $\Delta = M_{\Delta} - M_N \simeq 3f_{\pi}$, where M_{Δ} , M_N , and f_{π} denote the mass of the delta, of the nucleon, and the pion decay constant, respectively.¹ In the effective field theory sketched above this leads to unnaturally large values of some low–energy constants.

It is also possible to include the delta as dynamical degree of freedom in the effective field theory [4, 5]. For the πN system this leads to a somewhat improved convergence of the chiral expansion [6], however, no qualitative difference appears compared to ChPT. In many cases, the representation of delta effects through local pion-nucleon operators is quite accurate. As an example we mention the

¹Like the pion decay constant, the delta–nucleon mass splitting does not vanish in the chiral limit and thus this identification is more appropriate than $\Delta \simeq 2m_{\pi}$, with m_{π} the pion mass, as often found in the literature.

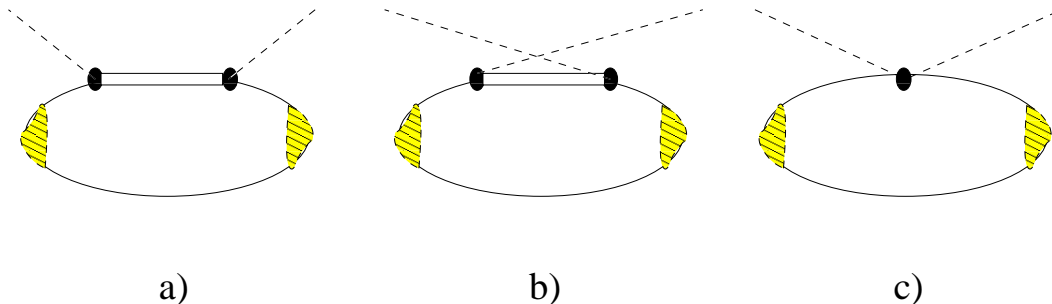


Figure 1: Classes of one-body diagrams that contribute to πd scattering. Diagrams a) and b) represent the one-body operators with the delta, diagram c) shows the corresponding contact interaction in the delta-less theory. Dashed lines denote pions and single (double) solid lines denote nucleons (deltas). Solid black dots stand for interactions, whereas the hatched area shows the deuteron wave function.

successful analysis of threshold pion photoproduction [7]. However, not only for energies of the order of Δ , but also at low energies in the spin sector the explicit inclusion of the delta appears mandatory — see, e.g., the recent review [8]. It is, however, important to stress that the delta-full theory in the single nucleon sector features more counter terms at a given order than ChPT and has been much less systematically applied to low-energy reactions.

In the present paper we investigate the role of the delta isobar in the reaction $\pi d \rightarrow \pi d$ at threshold in chiral effective field theory. The reason why the explicit inclusion of the delta in pion reactions on the two-nucleon system is beneficial is that the πN amplitudes appear in the boosted frame due to the Fermi motion of the nucleons. Let us for simplicity focus on the one-body terms with the delta that contribute to πd scattering at threshold (see Fig. 1a) and b)). Then the $\pi N \Delta$ transition vertex is linear in the nucleon momentum, \vec{p} , and the corresponding embedded $\pi N \rightarrow \pi N$ transition potential is proportional to \vec{p}^2 times the nucleon-delta propagator. The latter behaves as $1/(m_\pi - \Delta - \vec{p}^2/M_N)$ — we point out that the width of the delta is suppressed by two powers in the pion mass and thus does not contribute to the order we are working. For static deltas, this propagator reduces to $1/(m_\pi - \Delta)$ and the sum of diagrams a) and b) of Fig. 1 collapses to diagram c). Thus, in the latter case the transition operator behaves like \vec{p}^2 , whereas in the former it approaches a constant for momenta larger than $|\vec{p}_\Delta| \sim \sqrt{(\Delta - m_\pi)M_N} \sim 2.7m_\pi$ with the effect that the static amplitude is more sensitive to the short range part of the deuteron wave function and must be balanced by appropriate counter terms, eventually of unnatural size. The operator with the dynamical delta, on the other hand, does not share this problematic property. This point will be discussed in detail below. The value of p_Δ is numerically very close to $p_{\text{thr}} = \sqrt{M_N m_\pi} \sim 2.6m_\pi$ — the minimum initial momentum for the reaction $NN \rightarrow NN\pi$. Therefore, in what follows we will use

$$p_\Delta \sim p_{\text{thr}} \gg m_\pi . \quad (1)$$

It was shown in Ref. [9] that the so-called dispersive corrections to the πd scattering length are suppressed by a factor $\chi^{3/2}$ relative to the leading two-body operator with two Weinberg–Tomozawa (WT) vertices, where $\chi = m_\pi/M_N$. The corresponding power counting, confirmed numerically, treated explicitly the scale $p_{\text{thr}} \gg m_\pi$ in line with the counting rules for $NN \rightarrow NN\pi$ [10]. The counting rule Eq. (1) automatically puts the delta contributions in the same order as the dispersive corrections, as we demonstrate below. There is one more class of contributions that can scale as $\sqrt{\chi}$ in few-nucleon systems, namely the effect of πNN cuts. However, their impact on πd scattering is negligible as shown in Ref. [11]. Thus, with this paper we complete the calculation of diagrams at order $\chi^{3/2}$.

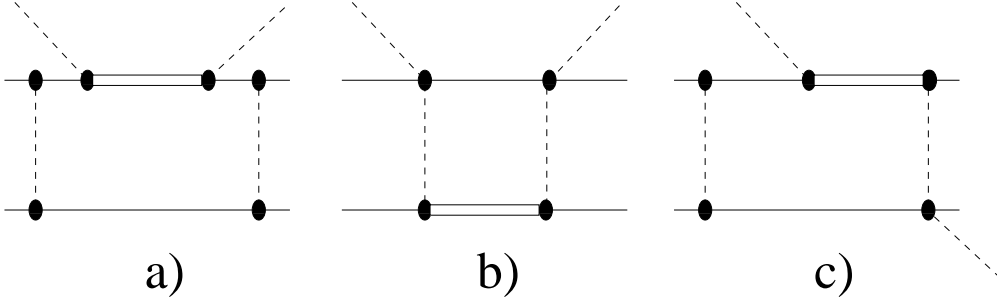


Figure 2: Typical subamplitudes that contribute to πNN scattering including deltas at order $\chi^{3/2}$.

The paper is structured as follows: in the next section we describe the power counting. Results for the πd scattering length are reported and compared to previous works in section 3. The paper ends with some concluding remarks.

2 Power counting

First of all we would like to remind the reader that the leading order two-body operator with two WT vertices scales simply as $m_\pi^2/(f_\pi^4 p^2) \sim 1/f_\pi^4$ in the Weinberg counting scheme, where for this diagram $p \sim m_\pi$. Below we will follow the logic of Ref. [9] and compare diagrams with the delta with this leading amplitude. Let us start with the one-body terms depicted in Fig. 1a). The transition amplitude scales as

$$A_{\pi N}^\Delta = \frac{1}{f_\pi^2} \left(\frac{m_\pi}{M_N} \right)^2 \frac{\vec{p}^2}{m_\pi - \Delta - \vec{p}^2/M_N + i\epsilon}. \quad (2)$$

As outlined in the Introduction, due to the squared momentum in the numerator, momenta of order $|\vec{p}_\Delta| \sim \sqrt{(\Delta - m_\pi)M_N}$ contribute to the full matrix element. Therefore the nucleon before (after) the pion absorption (emission) is off its mass shell by $-\vec{p}_\Delta^2/2M_N$, i.e. by about m_π . On the other hand, only on-shell amplitudes are physically meaningful and should be compared to each other. To find the corresponding chiral order we should therefore estimate the one loop diagram as shown in Fig. 2a)². The estimate for this diagram gives

$$\left[\left(\frac{p^2}{f_\pi^2 p^2} \right)^2 \left(\frac{M_N}{p^2} \right)^2 A_{\pi N}^\Delta \left(\frac{p^3}{(4\pi)^2} \right) \right] / \left(\frac{1}{f_\pi^4} \right) \sim \begin{cases} \mathcal{O}(\chi^2) & \text{for } p \sim m_\pi \\ \mathcal{O}(\chi^{\frac{3}{2}}) & \text{for } p \sim p_\Delta \end{cases} \quad (3)$$

where the factors stand for the quantitative estimates for the two one-pion exchange potentials, the two two-nucleon propagators, the $\pi N \rightarrow \pi N$ transition potential through the delta, as defined in Eq. (2), and the integral measure, consecutively. We stress that we do the power counting based on the expressions for time-ordered perturbation theory, since we later work within this formalism. For details on this we refer to Appendix E of Ref. [12]. In the relation (3) we estimated the contribution of diagram 2a) for two regimes of pion momenta, namely $p \sim m_\pi$ and $p \sim p_\Delta$. For the identification of the chiral order we used $M_N \sim 4\pi f_\pi$. We thus conclude that the power counting yields that the dominant contribution of the delta loops is expected to come from loop momenta of the order of p_Δ , as

²Note that the external nucleons in Fig. 2a) can also be off-shell, when the transition operators are convoluted with the external wave functions. However, it is the central assumption of the power counting that the corresponding matrix element is dominated by (near) on-shell kinematics for these nucleons.

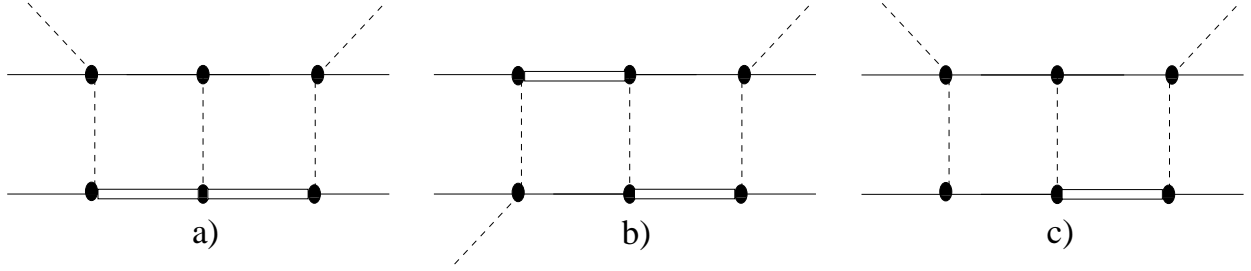


Figure 3: Typical subamplitudes that contribute to πNN scattering including deltas at order χ^2 . These (and all the others of such kind) are not included in this work.

argued above. Therefore the considered delta diagram contributes to the same order as the so-called dispersive corrections to the πd scattering length as discussed in Ref. [9].

The power counting for the other πd diagrams goes in just the same way. E.g., for the diagrams b) and c) of Fig. 2 we also find the chiral order $\chi^{3/2}$. The power counting for a diagram of type b), however with a two-nucleon intermediate state, was discussed in detail in Ref. [9]. Since we count the nucleon-delta propagator in the same manner as the two-nucleon propagator, i.e. as $1/m_\pi$, it becomes obvious that the corresponding diagrams contribute at the same order. In the estimation of the chiral order of diagrams b) and c) we used that the leading $\pi N \rightarrow \pi N$ vertex scales as m_π and not as the pion energy, regardless of the relatively large momentum running in the loop. The terms dropped are higher order in the chiral expansion. This is in line with the findings of Ref. [13] for the reaction $NN \rightarrow d\pi$. Every additional loop including deltas leads at least to an additional factor $p^3/(4\pi)^2 \times 1/m_\pi \times 1/f_\pi^2$ for the integral measure, the $N\Delta$ propagator and the leading $N\Delta$ interaction, consecutively. Therefore, diagrams with an intermediate $N\Delta \rightarrow N\Delta$ transition, as shown in Fig. 3a) and b), and those with an intermediate $NN \rightarrow N\Delta$ transition, diagram c), are suppressed by one power in $p/M_N \sim \chi^{1/2}$ compared to the diagrams shown in Fig. 2 and will not be considered in this work. Consequently, from the naive power counting arguments we can expect the leading delta contribution to be of order of $(m_\pi/M_N)^{3/2} |a_{\pi d}^{\text{double}}| \simeq 0.06 |a_{\pi d}^{\text{exp}}| \simeq 1.6 \times 10^{-3} m_\pi^{-1}$ where we used that $|a_{\pi d}^{\text{exp}}| \simeq 26 \times 10^{-3} m_\pi^{-1}$ and that the real part of the scattering length is dominated by the double rescattering term with two WT vertices — giving rise to $a_{\pi d}^{\text{double}}$. This estimation is fully in line with our numerical results as given in the next section. In addition, as stated already, we do not consider terms of order χ^2 . Using the same reasoning as above, we can also estimate the theoretical uncertainty of our calculation as

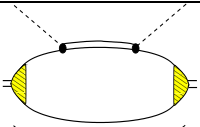
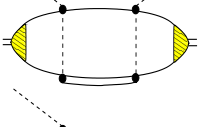
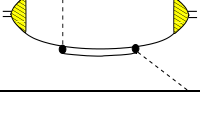
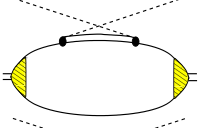
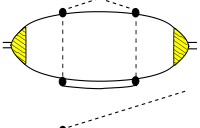
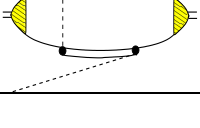
$$\Delta a^{\text{theor}} = (m_\pi/M_N)^2 |a_{\pi d}^{\text{double}}| \simeq 0.6 \times 10^{-3} m_\pi^{-1} . \quad (4)$$

At order χ^2 also the leading $NN\pi \rightarrow NN\pi$ counter term contributes to πd scattering with up-to-now unknown coefficient. Therefore Δa^{theor} represents at the same time an estimate for the theoretical accuracy for the extraction of the isoscalar scattering length a^+ from πd scattering [14]. For a further improvement in the accuracy of the calculation, input from other reactions is needed to fix the value of the counter term. One possible source of this information could be the reaction $NN \rightarrow NN\pi^0\pi^0$.

3 Results and comparison to previous works

Although the vertex structure we use for the $\pi N\Delta$ vertex is standard (see, e.g., Ref. [15] and references therein), in order to keep the paper self-contained and to fix the normalization we present it here (note

Table 1: Delta contributions to the real part of $a_{\pi d}$ in units of $m_{\pi}^{-1} \times 10^{-3}$ calculated with $h_A = 2.77$. The results are shown for the phenomenological NN potentials Paris [21], AV18 [22], CD-Bonn [23] (CDB), and CCF [24] as well as for the wave functions of the N²LO chiral NN interaction [25] based on the pairs of regulators $\{600, 500\}$ (EGM1), $\{550, 600\}$ (EGM2) and $\{450, 700\}$ (EGM3). All integrals are evaluated up to 1 GeV – the contributions of higher momenta are negligible.

		Paris	AV18	CDB	CCF	EGM1	EGM2	EGM3
$e1$	 =	+1.89	+1.92	+1.77	+1.81	+1.69	+1.66	+1.57
$e2$	 =	+0.54	+0.55	+0.73	+0.56	+0.72	+0.79	+0.84
$e3$	 =	-0.70	-0.73	-0.94	-0.72	-0.91	-1.02	-1.08
	sum of this group =	+1.73	+1.74	+1.56	+1.65	+1.50	+1.43	+1.33
$f1$	 =	+0.84	+0.85	+0.75	+0.79	+0.69	+0.67	+0.63
$f2$	 =	+0.13	+0.14	+0.21	+0.14	+0.21	+0.24	+0.26
$f3$	 =	-0.05	-0.05	-0.14	-0.06	-0.14	-0.19	-0.22
	sum of this group =	+0.92	+0.94	+0.82	+0.87	+0.76	+0.72	+0.67
	total sum =	+2.65	+2.68	+2.38	+2.52	+2.26	+2.15	+2.00

that our vertex normalization differs by a factor of two compared e.g. to the one of Ref. [16]):

$$\begin{aligned}\mathcal{L}^{(0)} &= \frac{h_A}{2f_\pi} [N^\dagger (\mathbf{T} \cdot \vec{S} \cdot \vec{\nabla} \boldsymbol{\pi}) \Psi_\Delta + \text{h.c.}] , \\ \mathcal{L}^{(1)} &= -\frac{h_A}{2M_\Delta f_\pi} [iN^\dagger \mathbf{T} \cdot \dot{\boldsymbol{\pi}} \vec{S} \cdot \vec{\nabla} \Psi_\Delta + \text{h.c.}] .\end{aligned}\tag{5}$$

Here h_A denotes the leading $\Delta N \pi$ coupling, and \vec{S} and \mathbf{T} are the spin and isospin transition matrices, normalized such that

$$\begin{aligned}S_i S_j^\dagger &= \frac{1}{3} (2\delta_{ij} - i\epsilon_{ijk} \sigma_k) , \\ T_i T_j^\dagger &= \frac{1}{3} (2\delta_{ij} - i\epsilon_{ijk} \tau_k) .\end{aligned}\tag{6}$$

In our calculations we use $f_\pi = 92.4$ MeV and $h_A = 3g_A/\sqrt{2} \simeq 2.1g_A = 2.77$, where $g_A = 1.32$ is the axial-vector coupling of the nucleon (derived from the Goldberger-Treiman relation). The relation between h_A and g_A can be derived from large N_c arguments and the resulting coupling gives a reasonable description of the delta width at tree level [17]. Very similar values were shown to be consistent with the πN phase shifts in the delta region [18, 19]. It should be noted, however, that the dispersion theoretical analysis of Ref. [20] leads to the considerably lower value of $h_A = 2.1$.

In Table 1 we show the results of our numerical calculations for the complete set of diagrams with the delta isobar that contribute at order $\chi^{3/2}$. These numbers were produced using our preferred value $h_A = 2.77$. In order to study the model dependence of the results we performed the calculations for various NN potentials. Note that we used phenomenological NN models without [21, 22, 23] and with [24] explicit delta degree of freedom, as well as three variants of NN wave functions derived within chiral effective field theory [25]. We remark that ideally one would also use chiral wave functions with explicit deltas. However, up to now corresponding wave functions of sufficient accuracy exist only for higher partial waves [16]. Using the different potentials mentioned, we obtain

$$\delta a_{\pi d}^\Delta = (2.38 \pm 0.40) \times 10^{-3} m_\pi^{-1} ,\tag{7}$$


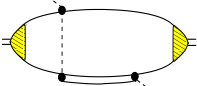
where the central value is the arithmetic average of the results for the seven different potentials and the uncertainty reflects the variations in the results. Consistency of the power counting demands that the dependence on the NN potential used does not exceed the contribution estimated for the leading counter term, Δa^{theor} given in Eq. (4), that can absorb this dependence. In this sense Eq. (7) is an additional confirmation for the consistency of the power counting employed.

All diagrams evaluated contain the $\pi N \Delta$ coupling constant h_A squared. Thus, to see the impact of a value as low as $h_A = 2.1$ on our results, the numbers given in Table 1 simply need to be rescaled. We then would get $\delta a_{\pi d}^\Delta|_{h_A=2.1} = (1.4 \pm 0.2) \times 10^{-3} m_\pi^{-1}$. However, we regard Eq. (7) as our main result, since the value employed for h_A can be extracted from fits to the πN system in the delta region based on calculations consistent with the one discussed here [18, 19].

Note that the results from the chiral wave functions are systematically lower than those from the phenomenological potentials, which might be a consequence of differences of the NN interactions at intermediate range. This finding does not come unexpected. However, calculations to higher orders are necessary to draw more firm conclusions.

The results we found depend only very weakly on the NN models used. In contrast to this, many previous works find a significant model dependence when using phenomenological parameterizations for some of the diagrams discussed above. For example, in Refs. [26, 27] diagram *e3* of Table 1 was included by replacing the delta propagator and vertices by the phenomenological πN p -wave

Table 2: Results for diagrams $e1$ and $e3$ evaluated with the static delta propagator in units of $10^{-3}m_\pi^{-1}$. Integrals are evaluated up to 1 GeV.

		Paris	AV18	CDB	CCF	EGM1	EGM2	EGM3
$e1$		+3.0	+3.1	+2.5	+2.8	+2.2	+2.1	+1.9
$e3$		-0.3	-0.4	-0.8	-0.4	-1.0	-1.3	-1.4

amplitude expressed in terms of the p -wave volumes c_0 and c_1 and evaluated in the boosted frame (this is called SP-interference term in Ref. [26]). The evaluated matrix element shows a significant model dependence for it scales with the deuteron wave function at the origin (for a more detailed study of the model dependence of this quantity see Ref. [28]). To illustrate how large the model dependence of the corresponding amplitude could be, we give in Table 2 the results for the diagram $e3$ calculated with the static Δ propagator. The results vary by more than a factor of four when different NN models are employed. The corresponding results for the diagram $e1$ (see Refs. [26, 27, 29] for the corresponding phenomenological calculations), also given in Table 2, differ by a factor 1.6. As stressed already in the Introduction, once the kinetic energy of the delta is kept in the propagator, as demanded by the power counting, the above problem disappears and almost model-independent results emerge (see lines $e1$ and $e3$ in Table 1). In an effective field theory calculation without explicit deltas, diagrams $e1$ and $f1$ were included effectively as so-called boost corrections [30]. The resulting contribution to the πd scattering length turned out to be quite sizable, namely $(3 - 5) \times 10^{-3} m_\pi^{-1}$, depending on the regulator used for the NN potential. Evidently, the spread in the results is well above the estimate of Eq. (4), which, again, is a consequence of dropping the kinetic energy of the delta isobar. In the theory without deltas the pertinent one-body operator scales with the square of the nucleon momentum and therefore the corresponding expectation value is proportional to the nucleon kinetic energy inside the deuteron — this quantity is strongly model-dependent [28], which indicates that the power counting in the delta-less theory requires further modification. However, the boost term (see Ref. [30]) is proportional to the low energy constant c_2 , which is known to be largely saturated by the delta isobar [31, 16]. In the analysis of the πN system [31] it was shown that the explicit evaluation of the leading order delta contribution results in a reduction of the value of c_2 from about 3.3 GeV^{-1} to about 0.5 GeV^{-1} . In the very recent analysis of the NN system including explicitly the delta at NLO [16], an analysis of πN threshold coefficients was performed. Given the parameters utilized there, the value of c_2 is reduced to -0.25 GeV^{-1} . A reduction of the $\pi N \Delta$ coupling by 30% as demanded by a dispersive analysis of the resonance contribution to the pion-nucleon P_{33} phase shifts [20] leads to a reduced $c_2 = 0.83 \text{ GeV}^{-1}$. All these values are consistent within the uncertainty of the various contributions to the low-energy constants given in Ref. [31]. Therefore the value of c_2 is reduced by a large factor once the delta contribution is taken out. We have calculated the residual boost correction using the expressions given in Ref. [30] with N²LO chiral wave functions and with $c_2 = -0.25 \text{ GeV}^{-1}$ and found it to be as small as $-(5.7 \dots 6.6) \times 10^{-4} m_\pi^{-1}$. Consequently, this correction is of the same size as the estimated uncertainty of the calculation (see. Eq. (4)) and thus does not contribute significantly anymore.

It should be stressed that it is not compulsory for a consistent calculation of the πd scattering length that the delta is included explicitly. Also a calculation without deltas is obviously equally justified. As usual the effects of the delta would then be parameterized by local counter terms of the type $\pi NN \rightarrow \pi NN$ with up-to-now unknown coefficients. The conclusion to be drawn from our studies is that in order to perform calculations with the accuracy of the order of the uncertainty estimate given in Eq. (4) it is necessary to include a dynamical delta, as long as no additional information on the size of the counter term is available. On the other hand, for a consistent inclusion of isospin breaking effects, that are known to be important [32], more theoretical work on the treatment of effects from quark masses and virtual photons in the delta-full theory would be useful.

4 Conclusions

In this work we calculated the leading contributions of the $\Delta(1232)$ to the πd scattering length in effective field theory. As expected, inclusion of the delta leads to an improved convergence for the isospin-symmetric operators that contribute to this reaction. We have also compared our results to other approaches and discussed the differences.

In the power counting employed the delta starts to contribute at order $\chi^{3/2}$, relative to the leading two-nucleon contribution, given by two subsequent πN scatterings on the two different nucleons. At the same chiral order the so-called dispersive corrections evaluated in Ref. [9] contribute as well, and with this work we complete the evaluation of diagrams at that order. In Ref. [9] the dispersive corrections were evaluated for a particular NN potential. When repeating the calculation with the four different phenomenological NN potentials employed in the present study (note: the chiral wave functions could not be used here, since for the dispersive corrections the wave functions are needed also at pion production threshold, where the chiral wave functions are not applicable anymore) we find

$$\delta a_{\pi d}^{\text{disp}} = (-2.9 \pm 1.4) \times 10^{-3} m_{\pi}^{-1}, \quad (8)$$

where the first number is the mean value for the various potentials and the second number reflects the theoretical uncertainty of this calculation estimated conservatively — see Ref. [9] for details. The variation of the results for the different potentials lies well within this uncertainty band. Note, that the uncertainty can be reduced by a calculation of $NN \rightarrow d\pi$ to next-to-next-to-leading order, which is planned for the near future. We therefore find for the total contribution at order $\chi^{3/2}$

$$\delta a_{\pi d}^{\Delta} + \delta a_{\pi d}^{\text{disp}} = (-0.6 \pm 1.5) \times 10^{-3} m_{\pi}^{-1}, \quad (9)$$

where we added the uncertainties given in Eqs. (7) and (8) in quadrature. Thus, we conclude that the net effect of the diagrams that contribute at order $\chi^{3/2}$ is very small. Note that the occurring cancellation is accidental because very different physics contributes to the two classes of diagrams.

One important consequence of our investigations is that once the delta isobar is treated dynamically, as it is done in this paper, the so-called boost corrections give rise to an insignificant contribution in the theoretical analysis of the πd scattering length. Furthermore, for the same reason the phenomenological inclusion of pion rescattering (the so-called SP interference term) through a boosted p -wave amplitude, used in Refs. [26, 27], is expected to yield a very small contribution, well within the theoretical uncertainty given here — see also the corresponding discussion in Ref. [9].

With this work all strong, isospin-symmetric contributions to the πd scattering length have been calculated to very high accuracy. In principle we could now extract the isoscalar πN scattering length, a^+ , directly from the πd scattering length, since

$$a_{\pi-d} = 2a^+ + \langle \text{few-body corrections } (a^-) \rangle, \quad (10)$$

where a^- denotes the isovector scattering length. In this expression additional terms that contain a^+ were neglected for they are numerically negligible. However, in addition isospin violating effects are known to be quite sizable. Therefore, in Eq. (10) we should replace $2a^+$ by $a_{\pi^-p} + a_{\pi^-n}$ which agrees to the former only, if isospin were an exact symmetry. Furthermore, few-body corrections involving virtual photons, in addition to those calculated in Ref. [9], are potentially important. For the π^-d system so far only the leading isospin violating corrections were evaluated [32]. To this order the largest theoretical uncertainty emerged from the appearance of the low-energy constants f_1 and c_1 . It is intriguing to observe, however, that those appear in the same linear combination in both a_{π^-p} and a_{π^-n} . Thus, one is in the position to extract $a_{\pi^-p} + a_{\pi^-n}$ with high accuracy from a combined analysis of pionic deuterium and pionic hydrogen even without detailed knowledge on f_1 (see also Ref. [33]). However, it remains to be seen if the corrections at next-to-leading order in isospin violation do not distort this picture. Corrections at this order for the π^-p system were evaluated in Refs. [34, 35] and turned out to be quite sizable, especially those that come from the pion mass difference. In order to push also the calculation for the πd system to a similar level of accuracy in isospin violation, the π^-n scattering amplitude as well as some virtual photon exchanges in the π^-d system are still to be calculated.

Acknowledgments

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Appendix

In this appendix we present the explicit expressions for the amplitudes given in Table 1. Note that in accordance with the power counting, we only keep those amplitudes that contain intermediate states with the nucleon, the delta and at most real pions. The calculation is done in time-ordered perturbation theory (TOPT). Especially, we dropped the so-called stretched boxes. The corresponding correction to the πd scattering length due to the delta isobar is

$$\delta a_{\pi d}^\Delta = a_{\pi d}^\Delta(q_0 = m_\pi) + a_{\pi d}^\Delta(q_0 = -m_\pi) \quad (\text{A.1})$$

where the first and second terms correspond to the direct and crossed diagrams of Table 1, respectively. Here

$$a_{\pi d}^\Delta(q_0) = -\frac{h_A^2 m_\pi^2}{48\pi f_\pi^6 (1+m_\pi/2M_N)} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{q_0 - \Delta - q^2/2M_{N\Delta}} (I_{3P_1} + I_{5P_1} + I_{5F_1}) \quad (\text{A.2})$$

where $M_{N\Delta} = M_N M_\Delta / (M_N + M_\Delta)$ and I_{2S+1L_J} are the partial wave amplitudes squared that correspond to the decomposed intermediate $N\Delta$ state

$$\begin{aligned} I_{3P_1} &= \frac{1}{9} \left[I_1^\Delta(q) - \frac{3}{2\sqrt{2}} I_2^\Delta(q) - \frac{2f_\pi^2}{M_\Delta} \left(u(q) + \frac{w(q)}{\sqrt{2}} \right) \right]^2, \\ I_{5P_1} &= \frac{5}{9} \left[I_1^\Delta(q) - \frac{3}{10\sqrt{2}} I_3^\Delta(q) - \frac{2f_\pi^2}{M_\Delta} \left(u(q) - \frac{w(q)}{5\sqrt{2}} \right) \right]^2, \\ I_{5F_1} &= \frac{3}{5} \left[I_4^\Delta(q) - \frac{2f_\pi^2}{M_\Delta} w(q) \right]^2, \end{aligned} \quad (\text{A.3})$$

with the I_i^Δ denoting the integrals that correspond to the overlap of the deuteron wave function ($u(q)$ and $w(q)$ for the S- and D-waves, respectively) with the one-pion-exchange operator

$$\begin{aligned}
I_1^\Delta(q) &= - \int \frac{d^3p}{(2\pi)^3} \frac{1 + (\vec{p} \cdot \vec{q})/q^2}{2\omega_{\vec{p}+\vec{q}}} \left(u(p) + \frac{w(p)}{\sqrt{2}} \right) \left(\frac{1}{P_1} + \frac{1}{P_2^\Delta} \right), \\
I_2^\Delta(q) &= - \int \frac{d^3p}{(2\pi)^3} \frac{1 - (\vec{p} \cdot \vec{q})^2/(p^2 q^2)}{2\omega_{\vec{p}+\vec{q}}} w(p) \left(\frac{1}{P_1} + \frac{1}{P_2^\Delta} \right), \\
I_3^\Delta(q) &= - \int \frac{d^3p}{(2\pi)^3} \frac{3 + 4(\vec{p} \cdot \vec{q})/q^2 + (\vec{p} \cdot \vec{q})^2/(p^2 q^2)}{2\omega_{\vec{p}+\vec{q}}} w(p) \left(\frac{1}{P_1} + \frac{1}{P_2^\Delta} \right), \\
I_4^\Delta(q) &= - \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{-1 - 3(\vec{p} \cdot \vec{q})/q^2 + 3(\vec{p} \cdot \vec{q})^2/(p^2 q^2) + 5(\vec{p} \cdot \vec{q})^3/(p^3 q^3)}{2\omega_{\vec{p}+\vec{q}}} w(p) \left(\frac{1}{P_1} + \frac{1}{P_2^\Delta} \right).
\end{aligned} \tag{A.4}$$

Here P_1 and P_2^Δ correspond to the TOPT components of the pion propagator

$$\begin{aligned}
P_1 &= q_0 - \omega_{\vec{p}+\vec{q}} - (p^2 + q^2)/2M_N, \\
P_2^\Delta &= -\omega_{\vec{p}+\vec{q}} - \Delta - p^2/2M_N - q^2/2M_\Delta
\end{aligned} \tag{A.5}$$

with $\omega_{\vec{q}} = \sqrt{\vec{q}^2 + m_\pi^2}$. The diagrams of Table 1 can be easily matched to the individual terms of Eqs. (A.2) and (A.3): the very last terms on the r.h.s. of each amplitude I_{2S+1L_J} in Eqs. (A.3), proportional to the deuteron wave functions squared, correspond to the diagrams of type 1 ($e1$ and $f1$), type 2 contains I_i^Δ amplitudes squared, whereas the interference terms of type 3 contain the rest. For the direct terms, labeled as e in Table 1, one needs to take $q_0 = m_\pi$ and for the crossed terms, labeled as f in that Table, $q_0 = -m_\pi$. Finally, we remark that all integrals are evaluated up to a sharp momentum cut-off of 1 GeV. All higher momentum contributions are negligible and anyway are to be absorbed in a counter term that is to be included at order χ^2 . Calculated with different wave functions demonstrates nice convergence.

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